Name: _____

Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. True **FALSE** The formula for the number of ways to place b indistinguishable balls into u distinguishable urns injectively if b > u is 0 not $\binom{u}{b}$.

Solution: If b > u, then $\binom{u}{b} = 0$ so both are valid.

2. True **FALSE** For any stable marriage problem, there is only one stable matching.

Solution: It is possible to have multiple valid pairings.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (4 points) How many ways can I buy 250 bubble teas from RareTea for an event if there are 8 different options to choose from?

Solution: There are 250 indistiguishable balls which are the bubble teas, and the boxes are the options. So there are $\binom{250+8-1}{250} = \binom{257}{250}$ ways to do this.

(b) (4 points) The most popular option is original milk tea. How many ways can I do this if I need at least 30 of that option and at least 10 of every other option?

Solution: First I buy 30 originals and 10 of all the other options. So I've bought 100 bobas and need to buy 150 more out of the 8 options. There are $\binom{150+8-1}{150} = \binom{157}{157}$ ways to do this.

(c) (2 points) Suppose men and women have the preferences $m_1: w_1 > w_3 > w_2, m_2: w_2 > w_1 > w_3, m_3: w_3 > w_2 > w_1$ and $w_1: m_1 > m_2 > m_3, w_2: m_2 > m_3 > m_1, w_3: m_3 > m_2 > m_1$. Is the matching $(m_1, w_1), (m_2, w_3), (m_3, w_2)$ stable?

Solution: This is not stable because m_3 prefers w_3 to w_2 and w_3 prefers m_3 to m_2 which means that both with divorce their partners and get with each other.