Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. True FALSE The formula for the number of ways to place $b$ indistinguishable balls into $u$ distinguishable urns injectively if $b>u$ is 0 not $\binom{u}{b}$.

Solution: If $b>u$, then $\binom{u}{b}=0$ so both are valid.
2. True FALSE For any stable marriage problem, there is only one stable matching.

Solution: It is possible to have multiple valid pairings.

Show your work and justify your answers. Please circle or box your final answer.
3. (10 points) (a) (4 points) How many ways can I buy 250 bubble teas from RareTea for an event if there are 8 different options to choose from?

Solution: There are 250 indistiguishable balls which are the bubble teas, and the boxes are the options. So there are $\binom{250+8-1}{250}=\binom{257}{250}$ ways to do this.
(b) (4 points) The most popular option is original milk tea. How many ways can I do this if I need at least 30 of that option and at least 10 of every other option?

Solution: First I buy 30 originals and 10 of all the other options. So I've bought 100 bobas and need to buy 150 more out of the 8 options. There are $\binom{150+8-1}{150}=\binom{157}{157}$ ways to do this.
(c) (2 points) Suppose men and women have the preferences $m_{1}: w_{1}>w_{3}>w_{2}, m_{2}$ : $w_{2}>w_{1}>w_{3}, m_{3}: w_{3}>w_{2}>w_{1}$ and $w_{1}: m_{1}>m_{2}>m_{3}, w_{2}: m_{2}>m_{3}>$ $m_{1}, w_{3}: m_{3}>m_{2}>m_{1}$. Is the matching $\left(m_{1}, w_{1}\right),\left(m_{2}, w_{3}\right),\left(m_{3}, w_{2}\right)$ stable?

Solution: This is not stable because $m_{3}$ prefers $w_{3}$ to $w_{2}$ and $w_{3}$ prefers $m_{3}$ to $m_{2}$ which means that both with divorce their partners and get with each other.

